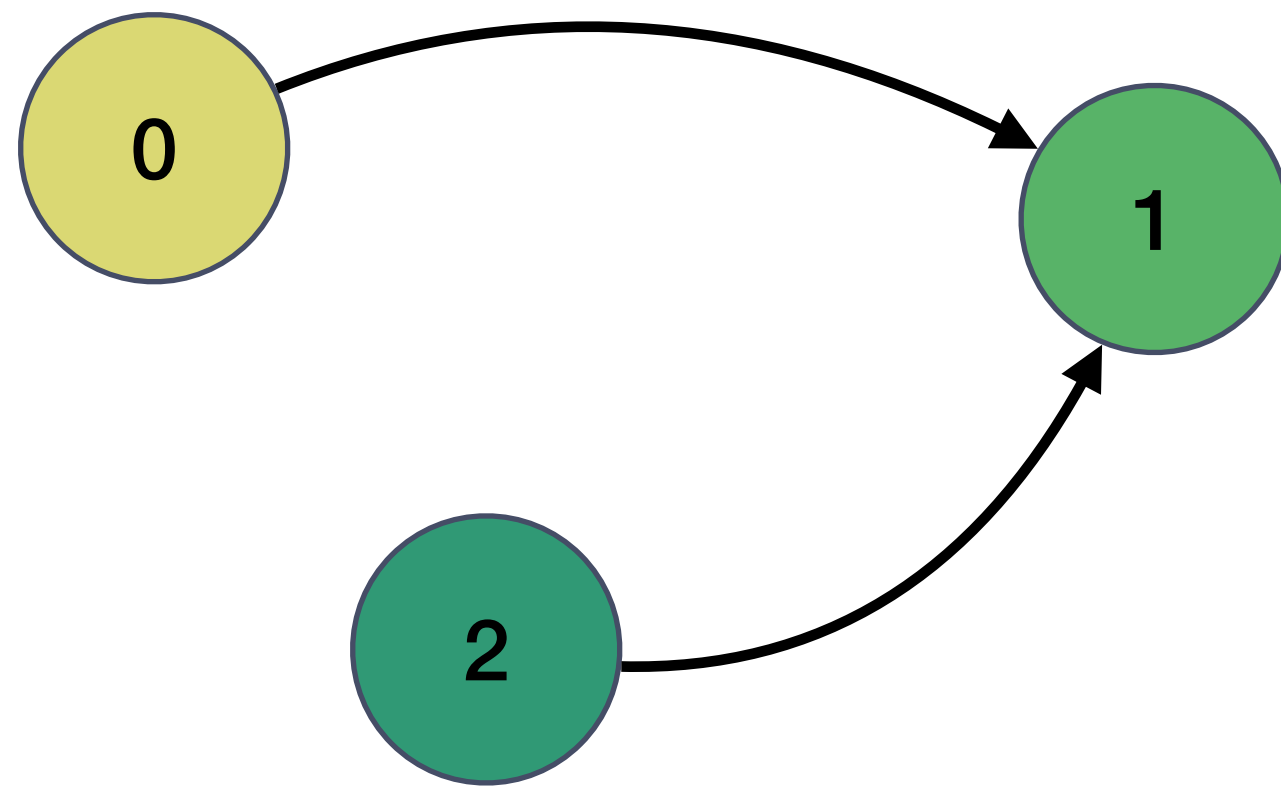


Dynamic graphs

Representation

Adjacency matrix



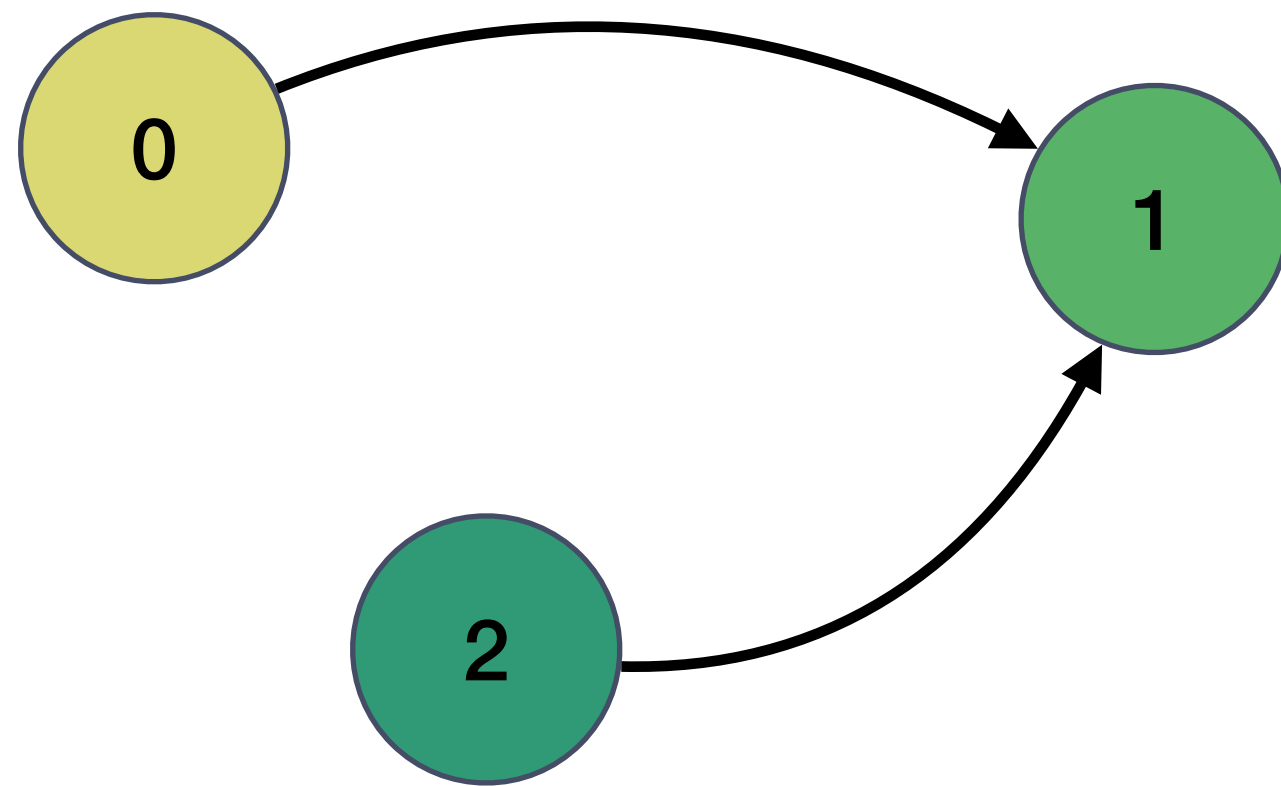
Destination

Source

$$\begin{bmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot \end{bmatrix}$$

Representation

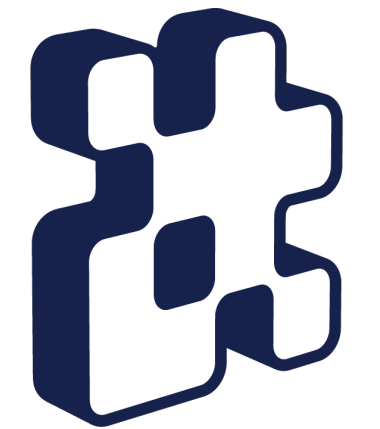
Adjacency matrix



Destination

Source

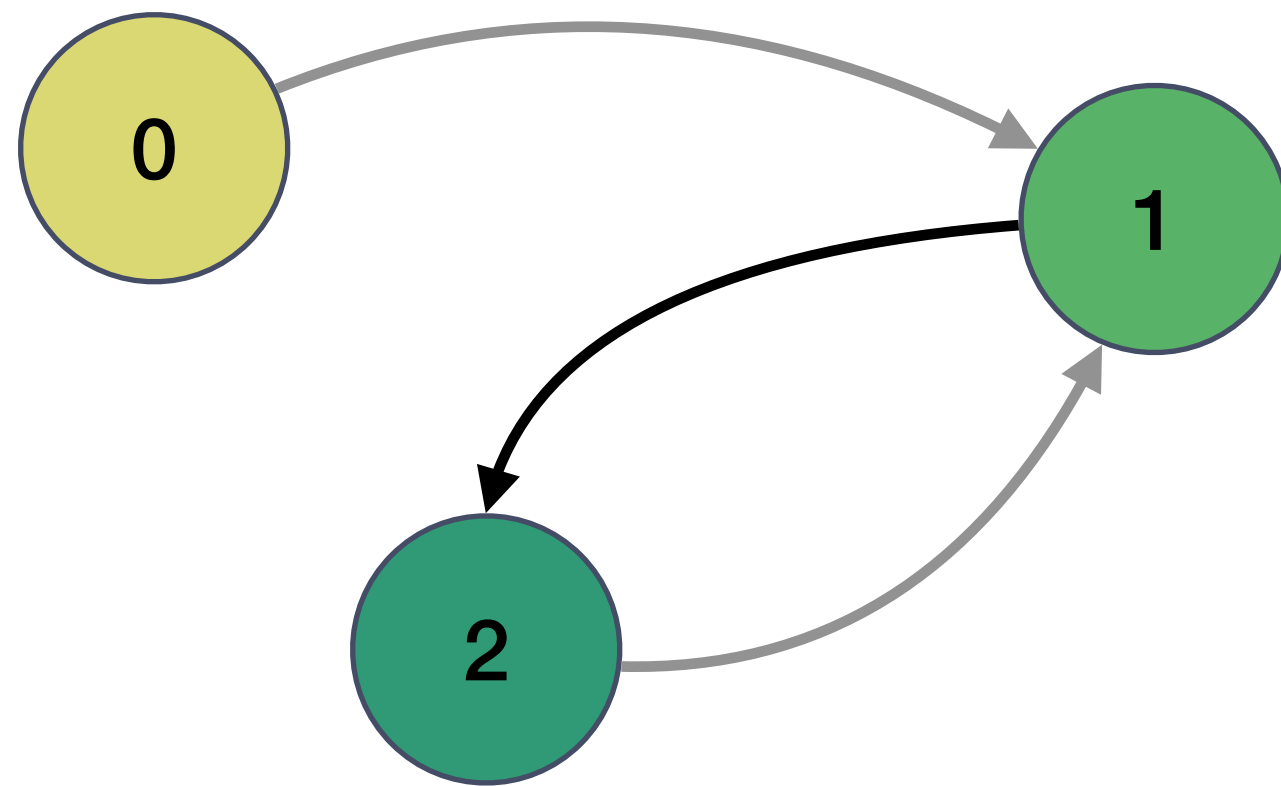
$$\begin{bmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot \end{bmatrix}$$



GRAPHBLAS

Set entry

Adjacency matrix



Destination

Source

$$\begin{bmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \\ \cdot & 1 & \cdot \end{bmatrix}$$



GRAPHBLAS

Worst case scenario

Read:Write

1:1

Benchmark

Sparse average update time

Dim	Avg ms
10K X 10K 500K	12.8
100K X 100K 4M	115.2
1M X 1M 30M	883.8
10M X 10M 50M	2415.7
80M X 80M 300M	17947.3

Set entry

CSR

Rows	<table border="1"><tr><td>0</td><td>2</td><td>5</td><td>6</td></tr></table> n+1	0	2	5	6		
0	2	5	6				
Columns	<table border="1"><tr><td>1</td><td>2</td><td>0</td><td>1</td><td>2</td><td>2</td></tr></table> nvals	1	2	0	1	2	2
1	2	0	1	2	2		
Values	<table border="1"><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr></table> nvals	1	1	1	1	1	1
1	1	1	1	1	1		

$$\begin{bmatrix} \cdot & 1 & 1 \\ 1 & 1 & 1 \\ \cdot & \cdot & 1 \end{bmatrix}$$

Set entry

CSR

Rows	0	2	5	6		
Columns	1	2	0	1	2	2
Values	1	1	1	1	1	1

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ \cdot & \cdot & 1 \end{bmatrix}$$

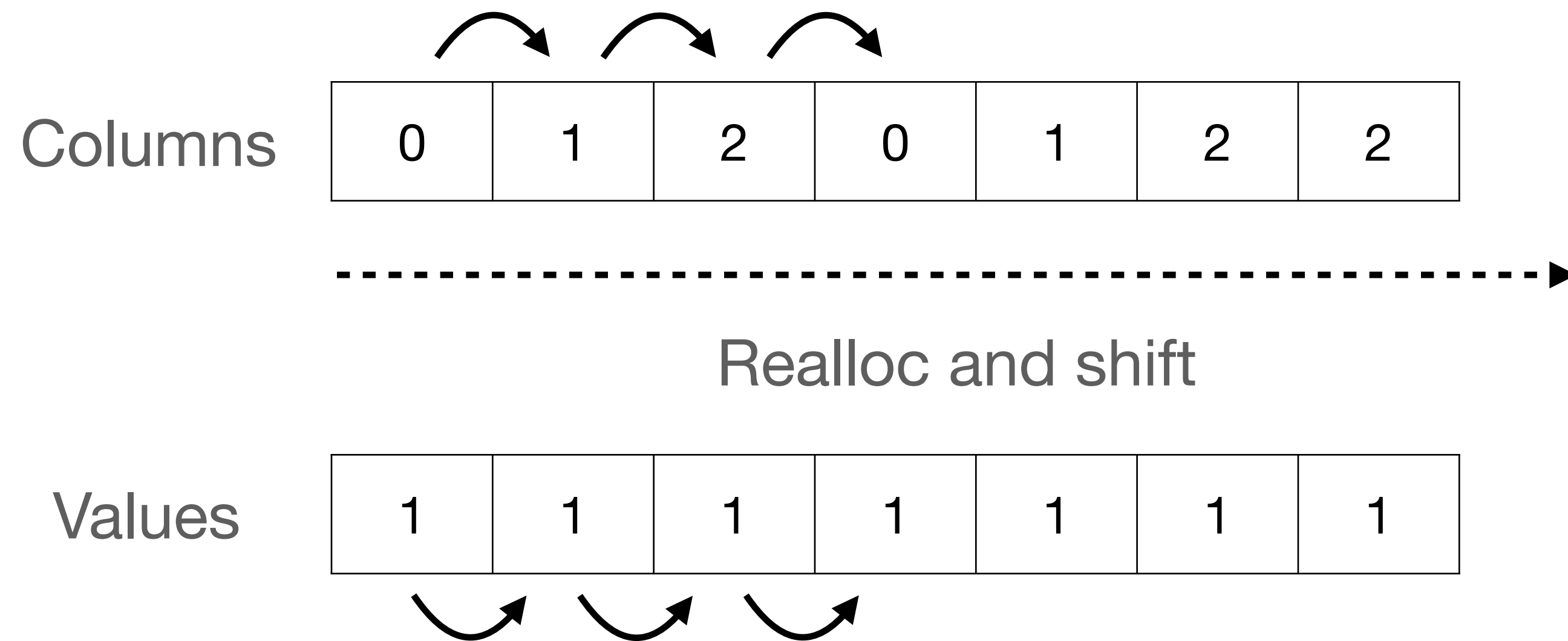
Set entry

Row adjustment

		+1	+1	+1
Rows	0	3	6	7

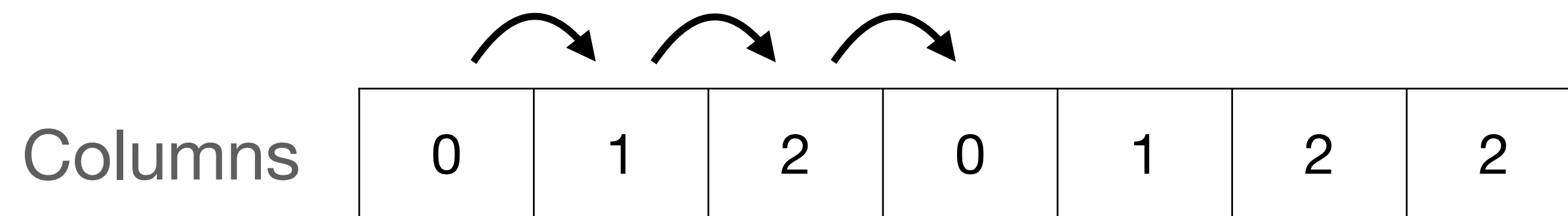
Set entry

Columns adjustment



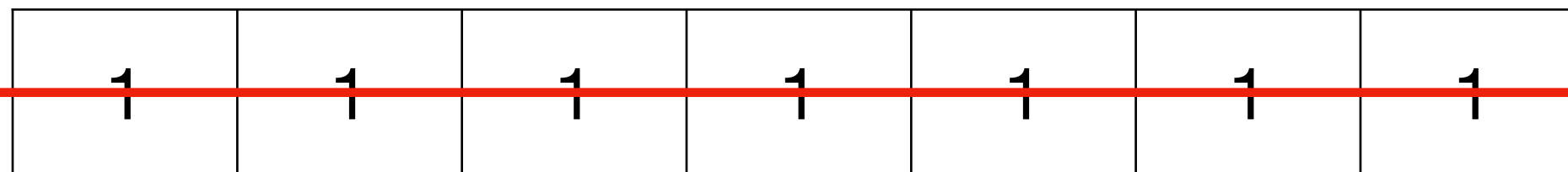
Set entry

Columns adjustment



Realloc and shift

~~Values~~



* ISO value

Set entry

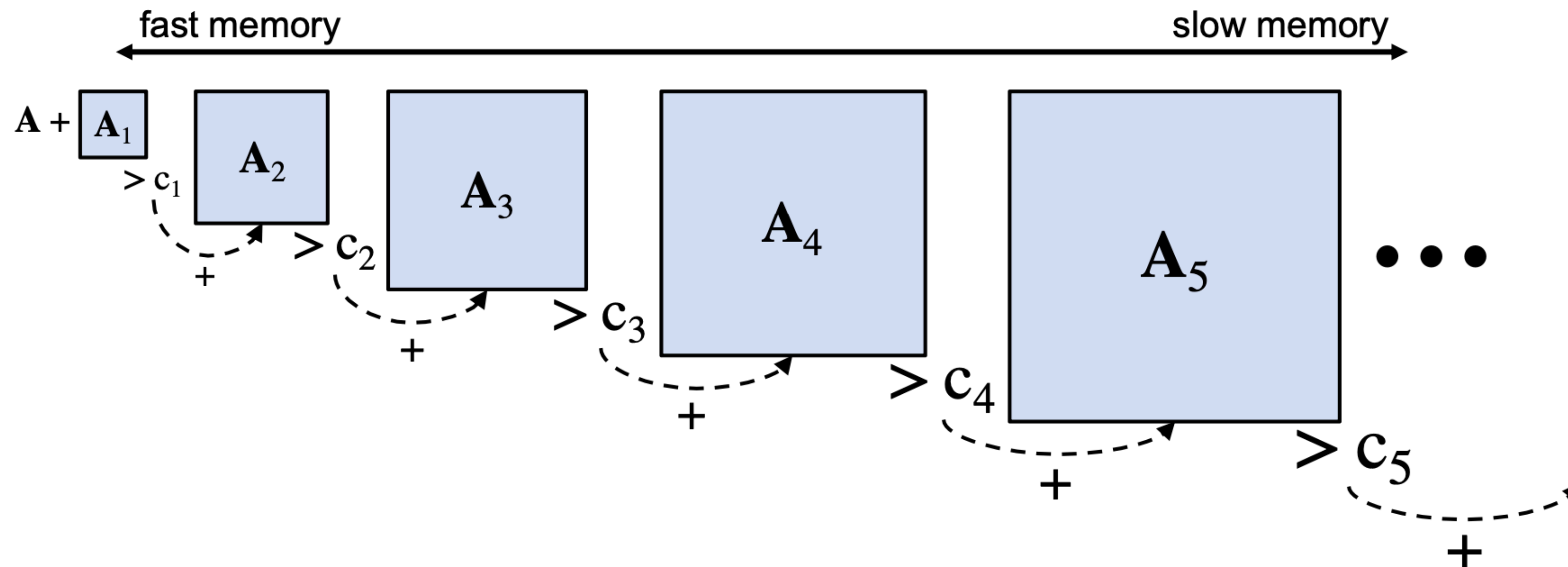
- Common operation
- Expensive
- Effected by matrix size

Vertical, Temporal, and Horizontal Scaling of Hierarchical Hypersparse GraphBLAS Matrices

Jeremy Kepner^{1,2,3}, Tim Davis⁴, Chansup Byun¹, William Arcand¹, David Bestor¹, William Bergeron¹, Vijay Gadepally^{1,2}, Michael Houle¹, Matthew Hubbell¹, Michael Jones¹, Anna Klein¹, Lauren Milechin⁵, Julie Mullen¹, Andrew Prout¹, Albert Reuther¹, Antonio Rosa¹, Siddharth Samsi¹, Charles Yee¹, Peter Michaleas¹

¹MIT Lincoln Laboratory Supercomputing Center, ²MIT Computer Science & AI Laboratory,

³MIT Mathematics Department, ⁴Texas A&M, ⁵MIT Department of Earth, Atmospheric and Planetary Sciences



Block representation

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Block representation

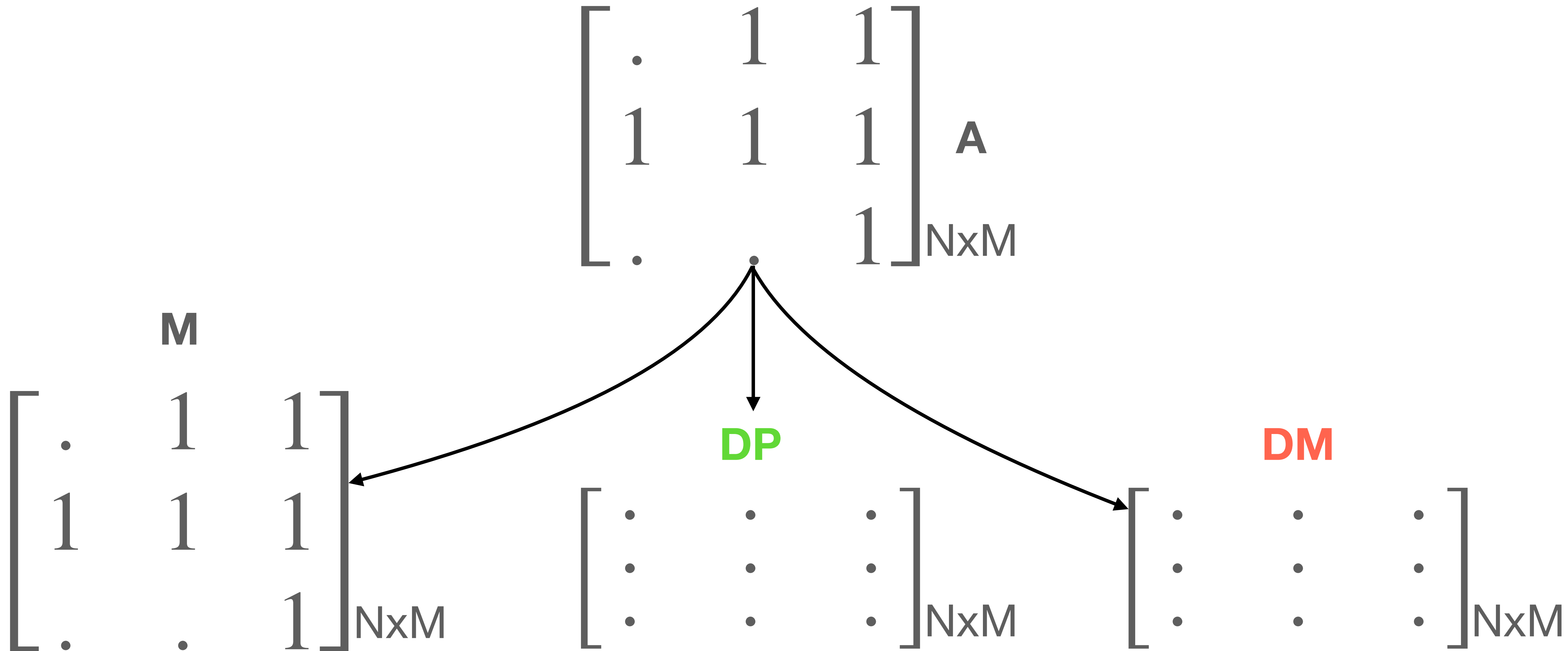
$$\begin{bmatrix} \begin{bmatrix} 1 & \cdot \\ \cdot & 1 \end{bmatrix} & \begin{bmatrix} \cdot & 1 \\ \cdot & \cdot \end{bmatrix} \\ \begin{bmatrix} 1 & \cdot \\ 1 & \cdot \end{bmatrix} & \begin{bmatrix} \cdot & 1 \\ 1 & \cdot \end{bmatrix} \end{bmatrix}$$

Delta matrix

$$\begin{bmatrix} \cdot & 1 & 1 \\ 1 & 1 & 1 \\ \cdot & \cdot & 1 \end{bmatrix}_{N \times M} \mathbf{A}$$

Delta matrix

Structure



Delta matrix

Structure

$$\begin{matrix} & & M \\ \begin{bmatrix} \cdot & 1 & 1 \\ 1 & 1 & 1 \\ \cdot & \cdot & 1 \end{bmatrix} \end{matrix}$$

- CSR
- **Immutable**

Delta matrix

Structure

$$\begin{bmatrix} \cdot & \text{DP} & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

- HYPER-SPARSE
- Capture recent additions

Delta matrix

Structure

$$\begin{bmatrix} \cdot & \text{DM} & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

- HYPER-SPARSE
- Capture recent deletions

Delta matrix

Set entry

$$A[0,0] = 1$$

M

$$\begin{bmatrix} \cdot & 1 & 1 \\ 1 & 1 & 1 \\ \cdot & \cdot & 1 \end{bmatrix}$$

DP

$$\begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

DM

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

Delta matrix

Del entry

DEL A[0,0]

M

$$\begin{bmatrix} . & 1 & 1 \\ 1 & 1 & 1 \\ . & . & 1 \end{bmatrix}$$

DP

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

DM

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

Delta matrix

Del entry

DEL A[1,1]

$$\begin{array}{c} \text{M} \\ \left[\begin{array}{ccc} \cdot & 1 & 1 \\ 1 & 1 & 1 \\ \cdot & \cdot & 1 \end{array} \right] \end{array}$$

$$\begin{array}{c} \text{DP} \\ \left[\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array} \right] \end{array}$$

$$\begin{array}{c} \text{DM} \\ \left[\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \boxed{1} & \cdot \\ \cdot & \cdot & \cdot \end{array} \right] \end{array}$$

Delta matrix

Set entry

$$A[1,1] = 1$$

M

$$\begin{bmatrix} \cdot & 1 & 1 \\ 1 & 1 & 1 \\ \cdot & \cdot & 1 \end{bmatrix}$$

DP

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

DM

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \boxed{\cdot} & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$|DP| + |DM| > T$$

Flush!

$$M = M \langle \sim \text{DM} \rangle + \text{DP}$$

Delta matrix

Flush

$$M = M_{\langle \sim DM \rangle} + DP$$

$$M \begin{bmatrix} . & 1 & 1 \\ 1 & 1 & 1 \\ . & . & 1 \end{bmatrix}$$

$$DP \begin{bmatrix} 1 & . & . \\ . & . & . \\ . & 1 & . \end{bmatrix}$$

$$DM \begin{bmatrix} . & 1 & 1 \\ 1 & . & . \\ . & . & . \end{bmatrix}$$

Delta matrix

Flush

DM

$$\begin{bmatrix} \cdot & 1 & 1 \\ 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

~DM

$$\begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Delta matrix

Flush

$M \langle \sim DM \rangle$

$$\begin{bmatrix} \cdot & \boxed{1} & \boxed{1} \\ \boxed{1} & 1 & 1 \\ \cdot & \cdot & 1 \end{bmatrix}$$

$\sim DM$

$$\begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Delta matrix

Flush

$$M = M_{\langle \sim \text{DM} \rangle} + \text{DP}$$

$$\begin{array}{c} M_{\langle \sim \text{DM} \rangle} \\ \left[\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{array} \right] \end{array} + \begin{array}{c} \text{DP} \\ \left[\begin{array}{ccc} 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot \end{array} \right] \end{array} = \begin{array}{c} M \\ \left[\begin{array}{ccc} 1 & \cdot & \cdot \\ \cdot & 1 & 1 \\ \cdot & 1 & 1 \end{array} \right] \end{array}$$

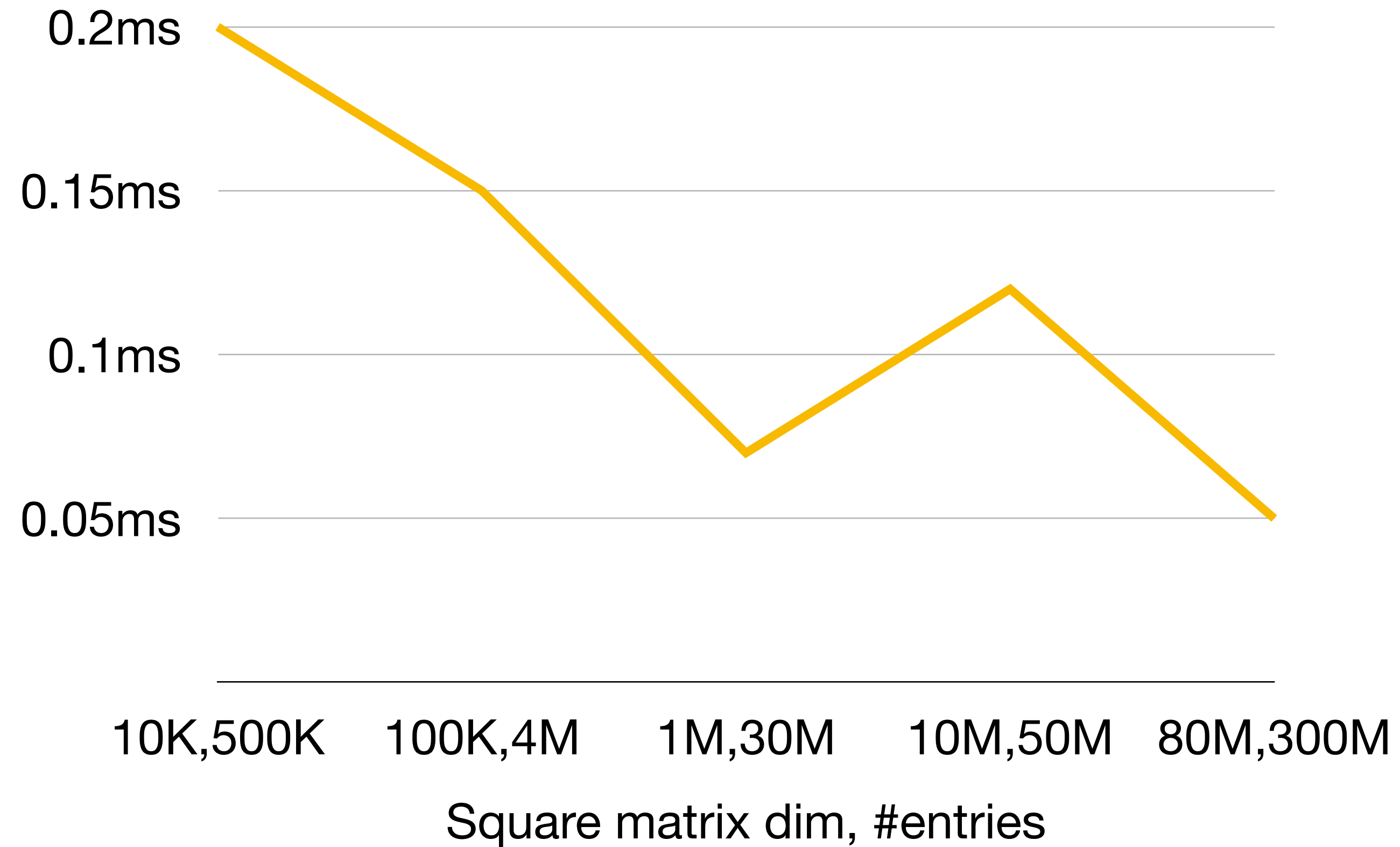
Benchmark

Sparse average update time

Dim	SPARSE Avg ms	DELTA Avg ms	Factor
10K X 10K 500K	12.8	0.2	64X
100K X 100K 4M	115.2	0.15	768X
1M X 1M 30M	883.8	0.07	12,625X
10M X 10M 50M	2415.7	0.12	20,130X
80M X 80M 300M	17947.3	0.05	358,946X

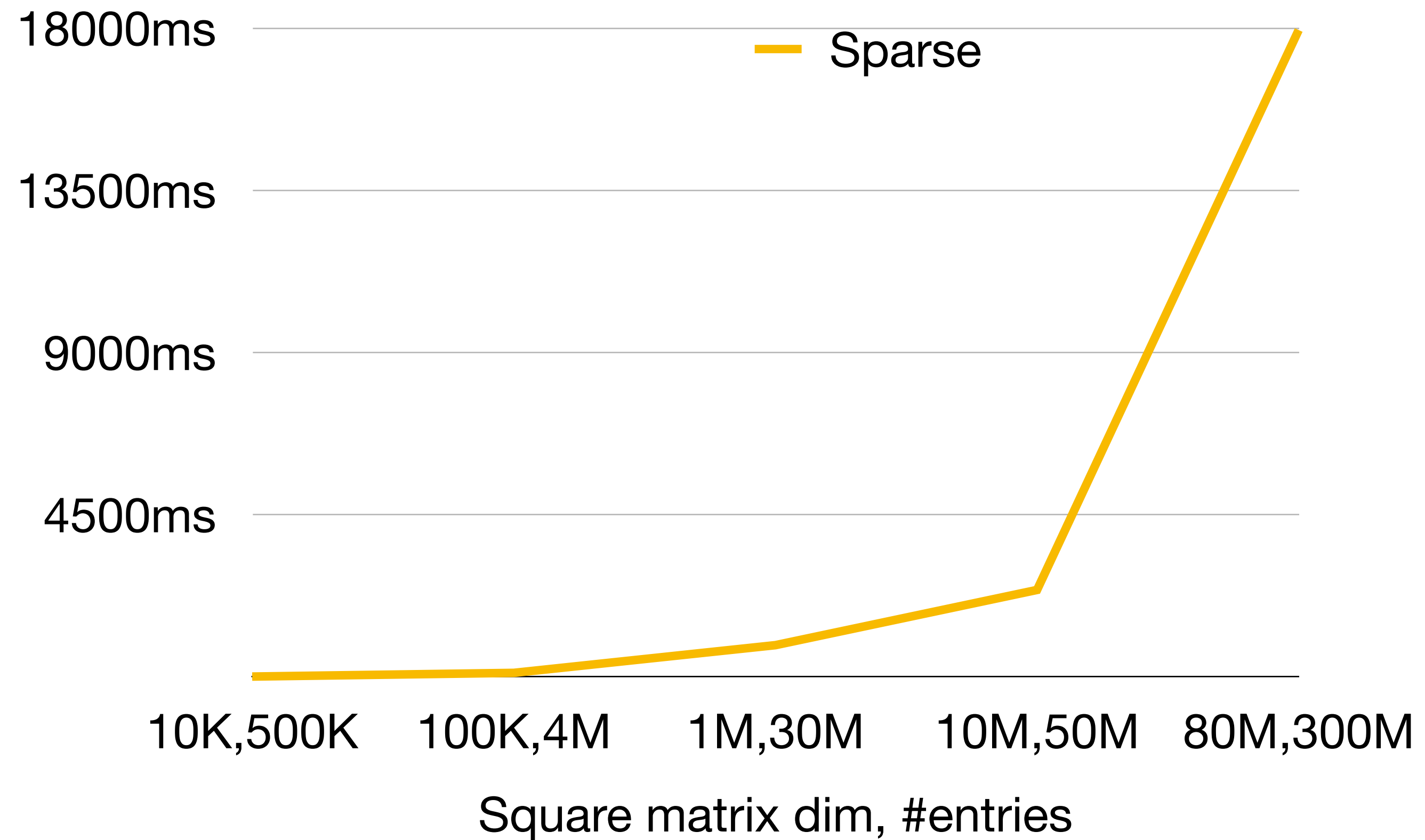
Benchmark

Delta, 5K 1:1 RW ops



Benchmark

Sparse / Delta 5K 1:1 RW ops



Links

[GraphBLAS](#)

[LAGraph](#)

[Benchmark](#)

Future work

- Block matrix
- MVCC

Thank you

roilipman@gmail.com